група 4

① $A = \left\{ \begin{bmatrix} x & 5y \\ 7y & x \end{bmatrix} : x,y \in R;\ x^2 \neq 35y^2 \right\}$

1° $A_{x,y} = \begin{bmatrix} x & 5y \\ 7y & x \end{bmatrix}$; $A_{u,v} = \begin{bmatrix} u & 5v \\ 7v & u \end{bmatrix}$

$$A_{x,y} \cdot A_{u,v} = \begin{bmatrix} xu+35yv & 5xv+5yu \\ 7yu+7xv & 35yv+xu \end{bmatrix} = \begin{bmatrix} \overset{\in R}{xu+35yv} & \overset{\in R}{5(xv+yu)} \\ 7(xv+yu) & xu+35yv \end{bmatrix}$$ 5

Пп. да је $(xu+35yv)^2 = 35(xv+yu)^2$, тј.

$x^2u^2 + 70\cancel{xy}uv + 35^2y^2v^2 = 35x^2v^2 + 70\cancel{xy}uv + 35y^2u^2$,

$x^2(u^2-35v^2) - 35y^2(u^2-35v^2) = 0$

$(x^2-35y^2)(u^2-35v^2) = 0$

Тада је $x^2-35y^2=0 \vee u^2-35v^2=0$, супротно претпоставци. 5

2° асоцијативност важи у општем случају 5

3°

$e = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in A$ јер је $1^2 \neq 35 \cdot 0^2$ 5

$A_{x,y} \cdot I = I \cdot A_{x,y} = A_{x,y}$; $\forall A_{x,y} \in A$.

4° $\det(A_{x,y}) = x^2 - 35y^2 \neq 0$

$cof(A_{x,y}) = \begin{bmatrix} x & -7y \\ -5y & x \end{bmatrix}$; $adj(A_{x,y}) = \begin{bmatrix} x & -5y \\ -7y & x \end{bmatrix}$

$A^{-1}_{x,y} = \begin{bmatrix} \frac{x}{x^2-35y^2} & 5\frac{-y}{x^2-35y^2} \\ 7\frac{-y}{x^2-35y^2} & \frac{x}{x^2-35y^2} \end{bmatrix} \in A$ 7 јер је $\frac{x^2}{(x^2-35y^2)^2} - 35 \cdot \left(\frac{-y}{x^2-35y^2}\right)^2 = \frac{1}{x^2-35y^2} \neq 0$ 3

$A^{-1}_{x,y}$ је инверзни елемент елемента $A_{x,y}$

5° $A_{u,v} \cdot A_{x,y} = \begin{bmatrix} u & 5v \\ 7v & u \end{bmatrix} \cdot \begin{bmatrix} x & 5y \\ 7y & x \end{bmatrix} = \begin{bmatrix} xu+35yv & 5yu+5xv \\ 7xv+7yu & 35yv+xu \end{bmatrix} = A_{x,y} \cdot A_{u,v}$ 5

Структура $(A, \cdot)$ је Абелова група.

· (5+5)
· 5
· 5
· 5+5
· 5

② a) $\left[\begin{array}{cccc|c} 2 & a & 3 & 1 & 1 \\ 1 & 3a-12 & 2 & 1 & 2 \\ 6 & 14 & 3a-7 & 2 & b-3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 3a-12 & 2 & 1 & 2 \\ 2 & a & 3 & 1 & 1 \\ 6 & 14 & 3a-7 & 2 & b-3 \end{array}\right] \begin{array}{c} (-2) \mid (-6) \\ + \\ \end{array} \sim$

$\sim \left[\begin{array}{cccc|c} 1 & 3a-12 & 2 & 1 & 2 \\ 0 & -5a+24 & -1 & -1 & -3 \\ 0 & -18a+86 & 3a-19 & -4 & b-15 \end{array}\right] \sim \begin{array}{c} \begin{array}{ccccc} x & u & z & y & \end{array} \\ \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3a-12 & 2 \\ 0 & -1 & -1 & -5a+24 & -3 \\ 0 & -4 & 3a-19 & -18a+86 & b-15 \end{array}\right] \end{array} (-4) \sim$

$\sim \begin{array}{c} \begin{array}{ccccc} x & u & z & y & \end{array} \\ \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3a-12 & 2 \\ 0 & -1 & -1 & -5a+24 & -3 \\ 0 & 0 & 3(a-5) & 2(a-5) & b-3 \end{array}\right] \end{array}$

1° $a \neq 5$: $r(A) = r(A^*) = 3$

$$\begin{aligned} x + u + 2z + (3a-12)y &= 2 \\ -u - z + (24-5a)y &= -3 \\ 3(a-5)z + 2(a-5)y &= b-3 \end{aligned}$$

$y = \alpha$:

$$\begin{aligned} x + u + 2z &= 2 - (3a-12)\alpha \\ -u - z &= -3 + (5a-24)\alpha \\ 3(a-5)z &= b-3-2(a-5)\alpha \end{aligned} \Rightarrow z = \frac{1}{3(a-5)} - \frac{2\alpha}{3}$$

$$-u = \frac{1}{3(a-5)} - \frac{2\alpha}{3} - 3 + (5a-24)\alpha$$

$$\boxed{u = 3 + \frac{2\alpha}{3} - \frac{1}{3(a-5)} - (5a-24)\alpha}$$

$$x = \frac{1}{3(a-5)} - \frac{2\alpha}{3} - 3 + (5a-24)\alpha - \frac{2}{3(a-5)} + \frac{4\alpha}{3} + 2 - (3a-12)\alpha \cancel{+2\alpha} = \boxed{-1 + \frac{2\alpha}{3} - \frac{1}{3(a-5)} + 2(a-6)\alpha}$$

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$$\boxed{\left\{\left(-1 + \frac{2\alpha}{3} - \frac{1}{3(a-5)} + 2(a-6)\alpha,\ \alpha,\ \frac{1}{3(a-5)} - \frac{2\alpha}{3},\ 3 + \frac{2\alpha}{3} - \frac{1}{3(a-5)} - (5a-24)\alpha\right)\right\}}$$

2° $a = 5$: $\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 & b-3 \end{array}\right]$

a) $b \neq 3$: $r(A) = 2 \neq r(A^*) = 3$; систем није сагласан 5

b) $b = 3$: $r(A) = r(A^*) = 2$

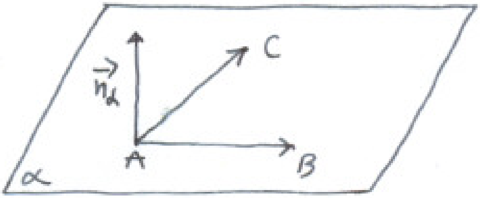
$$\begin{aligned} x + u + 2z + 3y &= 2 \\ -u - z - y &= -3 \end{aligned}$$

$z = \alpha$, $y = \beta$:

$$u = 3 - \alpha - \beta$$
$$x = 2 - 3 + \alpha + \beta - 2\alpha - 3\beta = -1 - \alpha - 2\beta$$

$\{(-1-\alpha-2\beta, \beta, \alpha, 3-\alpha-\beta) \mid \alpha, \beta \in R\}$ 12

③ a) $A(1,2,3)$; $B(4,-1,-2)$; $C(4,0,3)$



$\vec{AB} = (3,-3,-5)$

$\vec{AC} = (3,-2,0)$

$$\vec{n}_\alpha = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -3 & -5 \\ 3 & -2 & 0 \end{vmatrix} = -10\vec{i} - 15\vec{j} + 3\vec{k}$$

$\alpha: \quad -10(x-1) - 15(y-2) + 3(z-3) = 0$

$-10x - 15y + 3z + 31 = 0$ (10)

b) $|\vec{AB} \times \vec{AC}| = \sqrt{100+225+9} = \sqrt{334}$

$P_{\triangle ABC} = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{334}$ (7)

b) 10

c) 3