Zadaci iz predikatskog računa

Naći istinitosne vrednosti datih formula za zadate interpretacije:

1) $(\forall x)(\exists y)((\exists z)\alpha(f(x,z),f(y,z))\Rightarrow(\alpha(z,a)\Rightarrow\alpha(x,y)))$

$D=\mathrm{P}A,\ f:\cap,\ \alpha:=,\ a=\varnothing$, za $z=\varnothing$

Rad:

$(\forall x)(\exists y)((\exists z)\ x\cap z=y\cap z\Rightarrow(z=\varnothing\Rightarrow x=y))$

$(\forall x)(\exists y)((\exists z)\ x\cap z=y\cap z\Rightarrow(\varnothing=\varnothing\Rightarrow x=y))$

$\Phi(x,y)=((\exists z)\ x\cap z=y\cap z\Rightarrow(\varnothing=\varnothing\Rightarrow x=y))$

$v((\exists z)\ x\cap z=y\cap z)=1$ za $\forall x,y\in\mathrm{P}A$

$$v(\varnothing=\varnothing\Rightarrow x=y)=\begin{cases}1, & \text{za } x=y\\ 0, & \text{drugacije}\end{cases}$$

$$v(\Phi(x,y))=\begin{cases}1, & \text{za } x=y\\ 0, & \text{drugacije}\end{cases}$$

$v((\forall x)(\exists y)((\exists z)\ x\cap z=y\cap z\Rightarrow(\varnothing=\varnothing\Rightarrow x=y)))=1$

2) $(\forall x)((\exists z)\ \alpha(f(y,z),x)\Rightarrow(\exists y)\ \alpha(f(x,z),y))$

$D=N,\ \alpha:=,\ f:$ oduzimanje, za $y=2,\ z=3$

Rad:

$(\forall x)((\exists z)\ y-z=x\Rightarrow(\exists y)\,x-z=y)$

$(\forall x)((\exists z)\ 2-z=x\Rightarrow(\exists y)\,x-3=y)$

$\Phi(x,y)=((\exists z)\ 2-z=x\Rightarrow(\exists y)\,x-3=y)$

$$v((\exists z)\ 2-z=x)\Rightarrow\begin{cases}1, & \text{za } x<2\\ 0, & \text{drugacije}\end{cases}$$

$$v((\exists y)\,x-3=y)\Rightarrow\begin{cases}1, & \text{za } x>3\\ 0, & \text{drugacije}\end{cases}$$

$$v(\Phi(x,y))\Rightarrow\begin{cases}\left.\begin{array}{l}1,\ \text{za } x>3\\ 1,\ \text{za } x\geq 2\end{array}\right\}x\geq 2\\ 0,\ \text{drugacije}\end{cases}$$

$v((\forall x)\ \Phi(x,y))=0$

3) $(\forall z)(((\alpha(a,z)\Rightarrow\alpha(x,z))\vee(\alpha(x,y)\Rightarrow\alpha(y,z)))\wedge(\exists z)\alpha(x,f(y,z)))$

$D=\mathrm{P}A$, $\alpha:\subseteq$, $f:\cap$, $a=\varnothing$

Kolika je istinitosna vrednost ove formule za sve vrednosti promenljivih x i y za koje je $x\cap y\neq\varnothing$.

Rad:

$$\mathrm{P}(x,y)=(\forall z)(((\varnothing\subseteq z\Rightarrow x\subseteq z)\vee(x\subseteq y\Rightarrow y\subseteq z))\wedge(\exists z)x\subseteq y\cap z)$$

$$\Phi(x,y,z)=(((\varnothing\subseteq z\Rightarrow x\subseteq z)\vee(x\subseteq y\Rightarrow y\subseteq z))\wedge(\exists z)x\subseteq y\cap z)$$

Pošto $v(\varnothing\subseteq z)=1$ za $\forall z\in\mathrm{P}A$, tada

$$v(\varnothing\subseteq z\Rightarrow x\subseteq z)=\begin{cases}1, & x\subseteq z\\ 0, & \text{drugacije}\end{cases}$$

$$v(x\subseteq y\Rightarrow y\subseteq z)=\begin{cases}1, & y\subseteq z \text{ i } \forall x\in\mathrm{P}A\\ 1, & x\not\subseteq y \text{ i } \forall z\in\mathrm{P}A\\ 0, & \text{drugacije}\end{cases}$$

$$v((\exists z)\, x\subseteq y\cap z)=\begin{cases}1, & x\subseteq y\\ 0, & \text{drugacije}\end{cases}$$

$$v((\varnothing\subseteq z\Rightarrow x\subseteq z)\vee(x\subseteq y\Rightarrow y\subseteq z))=\begin{cases}1, & x\subseteq z \text{ i } \forall y\in\mathrm{P}A\\ 1, & y\subseteq z \text{ i } \forall x\in\mathrm{P}A\\ 1, & x\not\subseteq y \text{ i } \forall z\in\mathrm{P}A\\ 0, & \text{drugacije}\end{cases}$$

$$v(\Phi(x,y,z))=\begin{cases}1, & x\subseteq y \text{ i } x\subseteq z \;\; (*)\\ 1, & x\subseteq y \text{ i } y\subseteq z\\ 0, & \text{drugacije}\end{cases}$$

$$v((\forall z)\Phi(x,y,z))=\begin{cases}1, & x=\varnothing \text{ i } \forall y\in\mathrm{P}A\\ 0, & \text{drugacije}\end{cases}$$

Objašnjenje: Za $x=\varnothing$ važi $x\subseteq y$ za $\forall y$ i $x\subseteq z$ za $\forall z$ pa iz (*) sledi $v((\forall z)\Phi(\varnothing,y,z))=1$ za $\forall y$.

Ako je $x\cap y\neq\varnothing$, tada $x\neq\varnothing$ i $y\neq\varnothing$, pa je $v((\forall z)\Phi(x,y,z))=0$.

4) $(\forall x)(\exists y)(((\alpha(x,y)\vee\neg\alpha(y,z))\Rightarrow(\alpha(z,a))\Rightarrow(\exists z)\alpha(y,f(x,z)))$

$D=Z$, $\alpha:=$, $f:\text{mnozenje}$, $a=0$

Rad:

$P(z) = (\forall x)(\exists y)(((x = y \vee \neg y = z) \Rightarrow z = 0) \Rightarrow (\exists z)\ y = x \cdot z)$

$\Phi(x, y, z) = (((x = y \vee y \neq z) \Rightarrow z = 0) \Rightarrow (\exists z)\ y = x \cdot z)$

$$v(x = y \vee y \neq z) = \begin{cases} 1, & x = y \\ 1, & y \neq z \\ 0, & x \neq y \text{ i } y = z \end{cases}$$

$$v((x = y \vee y \neq z) \Rightarrow z = 0) = \begin{cases} 1, & z = 0 \text{ i } \forall x, y \in Z \\ 1, & x \neq y \text{ i } y = z \\ 0, & x = y \text{ i } z \neq 0 \\ 0, & y \neq z \text{ i } z \neq 0 \end{cases}$$

$$v((\exists z)\ y = x \cdot z) = \begin{cases} 1, & x \neq 0 \quad i \quad y \text{ deljivo sa } x \\ 1, & x = 0 \quad i \quad y = 0 \\ 0, & drugacije \end{cases}$$

$$v(\Phi(x, y, z)) = \begin{cases} 1, & x \neq 0 \quad i \quad y \text{ deljivo sa } x \quad i \quad \forall z \in Z \quad \} \ (*) \\ 1, & x = 0 \quad i \quad y = 0 \quad i \quad \forall z \in Z \quad \} \ (*) \\ 1, & x = y \quad i \quad z \neq 0 \\ 1, & y \neq z \quad i \quad z \neq 0 \\ 0, & drugacije \end{cases}$$

$v((\forall x)(\exists y)\Phi(x, y, z)) = 1$ za $\forall z \in \boldsymbol{Z}$ (sledi iz (*))

5) $(\exists x)(\forall y)(((\alpha(x,z) \Rightarrow \alpha(x,y)) \wedge \alpha(z,a)) \vee (\forall z)\alpha(x, f(y,z)))$

$D = PA,\ \alpha : =\ , f : \cup,\quad a = \emptyset,$ za $z = \emptyset$

Kolika je istinitosna vrednost formule za $z = A$?

Rad:

$(\exists x)(\forall y)(((x = z \Rightarrow x = y) \wedge z = \emptyset)) \vee (\forall z)\ x = y \cup z))$

$(\exists x)(\forall y)((x = \emptyset \Rightarrow x = y) \wedge \emptyset = \emptyset) \vee (\forall z)\ x = y \cup z)$

$\Phi(x, y) = ((x = \emptyset \Rightarrow x = y) \wedge \emptyset = \emptyset) \vee (\forall z)\ x = y \cup z)$

$$v((x = \emptyset \Rightarrow x = y) \wedge \emptyset = \emptyset) = \begin{cases} 1, x = y \\ 1, x \neq \emptyset \text{ i } \forall y \in PA \\ 0, \text{drugacije} \end{cases}$$

$$v((\forall z)\ x = y \cup z) = \begin{cases} 1, x = A \text{ i } y = A \\ 0, \text{drugacije} \end{cases}$$

$$v(\Phi(x,y)) = \begin{cases} 1, \; x = A \text{ i } y = A \\ 1, \; x = y \\ 1, \; x \neq \varnothing \text{ i } \forall y \in PA \quad (*) \\ 0, \; \text{drugacije} \end{cases}$$

$v((\exists x)(\forall y))\Phi(x,y)) = 1$ (sledi iz (*))

Za *z*=*A* važi da je $v((x = \varnothing \Rightarrow x = y) \wedge A = \varnothing) = 0$, pa je

$$v(\Phi(x,y)) = \begin{cases} 1, \; x = A \text{ i } y = A \\ 0, \; \text{drugacije} \end{cases} .$$

Zato je $v((\exists x)(\forall y))\Phi(x,y)) = 0$.

6) $(\exists x)((\exists z)\alpha(f(x,z),y) \vee (\neg\alpha(z,a) \wedge \alpha(x,y)))$

$D = \mathrm{P}A, \; \alpha := \; , f : \cup, \quad a = A,$ za $\quad y = \varnothing$

<u>Rešenje</u>: 1 za $\forall z \in \mathrm{P}A$

7) $(\forall y)(\forall x)((\exists z)\alpha(f(x,z), f(y,z)) \Rightarrow ((\neg\alpha(a,z) \Rightarrow \alpha(x,y)) \vee \alpha(a,z)))$

$D = \mathrm{R}, \; \alpha := \; , \; f : \text{mnozenje}, \quad a = 0$

<u>Rešenje</u>: 1 za *z*=0, 0 drugačije

8) $(\forall x)(\forall y)((\exists z)\alpha(f(x,z), f(y,z)) \Leftrightarrow ((\neg\beta(a,z) \Rightarrow \alpha(x,y)) \vee \beta(z,a)))$

$D = \mathrm{P}A, \; \alpha : \subseteq, \; \beta := , f : \cup, \quad a = A,$

<u>Rešenje</u>: 1 za *z*=*A*, 0 drugačije

9) $(\forall x)((\exists y)\alpha(f(x,y),x) \Leftrightarrow \alpha(y,a))$

$D = \mathrm{R}, \; \alpha := \; , \; f : \text{deljenje}, \quad a = 1$

<u>Rešenje</u>: 1 za *y*=1, 0 drugačije

10) $(\forall x)((\exists z)\alpha(x, f(y,z)) \Rightarrow (\exists y)\alpha(f(x,z), y))$

$D = \mathrm{N}, \; \alpha := \; , \quad f : \text{mnozenje},$

<u>Rešenje</u>: 1 za $\forall y, z \in \mathrm{N}$

11) $(\forall x)((\exists y)\alpha(f(x,y),x) \Leftrightarrow (\alpha(y,a) \vee \alpha(x,y))$

$D = \mathrm{P}A,\ \alpha :=\ ,f : \cup,\quad a = \varnothing,$ za $\quad y = A$

Rešenje: 0

12) $(\exists x)((\forall y)\alpha(f(x,y),x) \Leftrightarrow (\alpha(y,a) \vee \alpha(x,y))$

$D = \mathrm{P}A,\ \alpha :=\ ,f : \cup,\quad a = \varnothing,$

Rešenje: 1 za $\forall y \in \mathrm{P}(A)$

13) $(\forall y)((\exists x)\alpha(f(x,z),y) \Rightarrow (\neg\alpha(y,a) \vee \alpha(x,y)))$

$D = \mathrm{N},\ \alpha :=\ ,\ f : \text{mnozenje},\quad a = 4$

Rešenje: 0 za $\forall x \in \mathrm{N}$

14) $(\exists x)((\alpha(z,a) \wedge (\alpha(x,y) \Rightarrow \alpha(x,z))) \Rightarrow ((\exists z)\alpha(f(x,z)y) \vee \alpha(y,z)))$

$D = \mathrm{R},\ \alpha :=\ ,\ f : \text{mnozenje},\quad a = 0,\quad \text{za } z = 0.$

Rešenje: 1 za $\forall y \in \mathrm{R}$

15) $(\exists z)((\alpha(z,a) \vee (\alpha(x,y) \Rightarrow \alpha(x,z))) \Rightarrow ((\exists z)\alpha(f(x,z)y) \wedge (\alpha(y,z)))$

$D = \mathrm{PA},\ \alpha :=\ ,\quad f : \cap,\quad a = \varnothing$

Rešenje: 1 za $\mathrm{x} = \mathrm{y}$, 0 drugacije

16) $(\exists z)(((\forall z)\alpha(y, f(x,z)) \Rightarrow \alpha(z,a)) \wedge (\alpha(y,z) \vee \alpha(x,z)))$

$D = \mathrm{P}A,\ \alpha :=\ ,\quad f : \cup,\quad a = \varnothing$

Kolika je istinitosna vrednost formule za $x = y = A$?

Rešenje: 1 za $x \neq A$ i $\forall y \in \mathrm{P}A$

1 za $y \neq A$ i $\forall x \in \mathrm{P}A$

0 drugačije

Za $x = y = A$ istinitosna vrednost je 0.

17) $(\exists x)(\forall y)((\exists z)\alpha(f(x,z),x) \Rightarrow ((\neg\alpha(x,y) \wedge \alpha(y,z)) \Rightarrow \alpha(z,a)))$

$D = \mathrm{P}A,\quad f : \cap,\ \alpha :=,\ a = \varnothing,$ za $z = A$

Rešenje: 1 za $z = \varnothing,\ 0$ drugačije

18) $(\exists x)(\forall y)((\alpha(z,a)) \Rightarrow (\alpha(x,z) \vee \neg\alpha(x,y))) \Rightarrow (\exists z)\alpha(f(x,z),y))$

$D = \mathrm{P}A, \quad \alpha := , f : \cup , \quad a = \varnothing$

Rešenje: 1 za $\forall z \in \mathrm{P}A$

19) $(\forall z)(((\exists z)\,\alpha(y, f(x,z)) \wedge \alpha(y,z)) \Rightarrow ((\alpha(z,a) \Rightarrow \alpha(x,y)) \vee \alpha(x,z)))$

$D = \mathrm{P}A, \quad \alpha := , f : \cap , \quad a = A$

Rešenje: 1 za $y \not\subset x$, 0 drugačije.

20) $(\forall z)((\alpha(z,a) \wedge (\alpha(x,z) \Rightarrow \alpha(x,y))) \Rightarrow ((\exists z)\alpha(x, f(y,z)) \vee \alpha(y,z)))$

$D = N, \quad \alpha := , f :$ množenje, $\quad a = 1, \quad$ za $x = 3$ i $y = 2$

Rešenje: 0

21) $(\exists y)(\forall x)((\forall z)\,\alpha(f(x,z), y) \Rightarrow (\alpha(z,a) \Rightarrow (\alpha(x,y) \vee \neg\alpha(y,z))))$

$D = R , \quad \alpha := , f :$ množenje, $\quad a = 0$

Rešenje: 1 za $\forall z \in R$.

22) $(\exists x)(\forall y)((\exists z)\,\alpha(f(x,z), y) \Rightarrow ((\alpha(x,y) \wedge \neg\alpha(y,z)) \Rightarrow \alpha(z,a)))$

$D = \mathrm{P}A, \quad \alpha := , f : \cup , \quad a = \varnothing$

Rešenje: 1 za $z = \varnothing$, 0 drugačije.

23) $(\forall x)(\exists y)(((\forall z)\,\alpha(x, f(y,z)) \wedge \alpha(x,z)) \Rightarrow (\alpha(x,y) \Rightarrow \alpha(y,z)))$

$D = \mathrm{P}A , \quad \alpha := , f : \cap$

Rešenje: 1 za $\forall z \in \mathrm{P}A$.

24) $(\forall x)(\exists y)(((\alpha(x,z) \Rightarrow \alpha(x,y)) \Rightarrow \alpha(z,a)) \wedge (\exists z)\alpha(x, f(y,z)))$

$D = R , \quad \alpha := , \ \mathrm{a} = 0, f :$ sabiranje, za z=1.

Rešenje: 0

25) $(\exists x)(\forall y)((\exists z)\,\alpha(f(y,z), x) \Rightarrow ((\neg\alpha(x,y) \wedge \alpha(y,z)) \Rightarrow \alpha(z,a)))$

$D = N, \ f :$ mnozenje , $\alpha : <, \ a = 3$, za z=5.

Rešenje: 1

26) $(\exists x)(\forall y)((\alpha(z,a) \Rightarrow (\alpha(x,z) \Rightarrow \alpha(x,y))) \wedge (\exists z)\alpha(x, f(y,z)))$

$D = PA,\ a = A\ ,\ f : \cap\ ,\ \alpha := ,$ za z=A.

Rešenje: 1

27) $(\forall x)(\exists y)((\alpha(z,a) \wedge (\alpha(x,z) \Rightarrow \alpha(x,y))) \Rightarrow ((\exists z)\alpha(x, f(y,z)) \wedge \alpha(y,z)))$

$D = R,\ a = 0\ ,\ f : mnozenje\ ,\ \alpha := ,$ za z=0.

Rešenje: 0

28) $(\forall x)((\exists z)\alpha(f(x,z), y) \Rightarrow (\alpha(z,a) \Rightarrow (\neg\alpha(x,y) \vee \alpha(y,z))))$

$D = \mathrm{P}A\ , \alpha := ,\ f : \cap\ ,\ a = A,$ za $y = \varnothing$ i $z = A$.

Rešenje: 0

29) $(\forall x)(\exists y)(((\alpha(x,z) \Rightarrow \alpha(x,y)) \Rightarrow \alpha(z,a)) \wedge (\exists z)\alpha(x, f(y,z)))$

$D = \mathrm{P}A\ , \alpha := ,\ f : \cap\ ,\ a = \varnothing,$ za z=A.

Rešenje: 0

30) $(\exists x)(\forall y)(((\alpha(x,z) \Rightarrow \alpha(x,y)) \wedge \alpha(z,a)) \vee (\forall z)\alpha(x, f(y,z)))$

$D = R,\ \alpha := ,\ f : \text{mnozenje},\ a = 0\ ,$ za z=0.

Rešenje: 0