1. пр. $\displaystyle\int \frac{2\ln^2 x + 5\ln x + 2}{x(\ln x - 2)(\ln^2 x + 4\ln x + 8)}\,dx$

1. СМЕНА

$\left[\; t = \ln x,\quad dt = \frac{dx}{x} \;\right]$ 5

$$= \int \frac{2t^2 + 5t + 2}{(t-2)(t^2+4t+8)}\,dt$$

$$= \int \frac{1}{t-2}\,dt + \int \frac{t+3}{t^2+4t+8}\,dt$$

$$= \int \frac{1}{t-2}\,dt + \int \frac{(t+2)+1}{t^2+4t+8}\,dt$$

$$= \ln|t-2| \;③ + \int \frac{t+2}{t^2+4t+8}\,dt \;① + \int \frac{1}{t^2+4t+8}\,dt$$

2 СМЕНА:

$\left[\; y = t^2+4t+8,\quad dy = (2t+4)\,dt,\quad \frac{dy}{2} = (t+2)\,dt \;\right]$

$$= \ln|t-2| + \int \frac{\frac{dy}{2}}{y} \;③ + \int \frac{dt}{(t+2)^2 + 2^2}$$

$$= \ln|t-2| + \frac{1}{2}\ln|y| \;② + \frac{1}{4}\int \frac{dt}{\left(\frac{t+2}{2}\right)^2 + 1}$$

3. СМЕНА:

$\left[\; \frac{t+2}{2} = z,\quad dt = 2dz \;\right]$

$$= \ln|t-2| + \frac{1}{2}\ln(t^2+4t+8) + \frac{1}{4}\int \frac{2dz}{z^2+1}$$

$$= \ln|t-2| + \ln\sqrt{t^2+4t+8} + \frac{1}{2}\operatorname{arctg} z \;③ + C$$

$$= \ln|\ln x - 2| + \ln\sqrt{\ln^2 x + 4\ln x + 8} + \frac{1}{2}\operatorname{arctg}\left(\frac{\ln x + 2}{2}\right) \;③ + C \;②$$

$$\frac{2t^2+5t+2}{(t-2)(t^2+4t+8)} = \frac{A}{t-2} + \frac{Bt+C}{t^2+4t+8} \quad ⑤$$

$$= \frac{At^2 + 4At + 8A + Bt^2 + Ct - 2Bt - 2C}{(t-2)(t^2+4t+8)}$$

$$= \frac{(A+B)t^2 + (4A-2B+C)t + 8A - 2C}{(t-2)(t^2+4t+8)}$$

$$\begin{aligned} A+B &= 2 \\ 4A-2B+C &= 5 \\ 8A-2C &= 2 \;/\cdot\left(+\tfrac{1}{2}\right) \end{aligned} \quad + \qquad \left.\begin{aligned} A+B &= 2 \\ 8A-2B &= 6 \end{aligned}\right\} \rightarrow \begin{aligned} A &= 1 \\ B &= 1 \\ C &= 3 \end{aligned} \quad ⑧$$

•) Израчунати запремину тела насталог ротирањем криве $y=\sqrt{\sqrt{x}\sin\sqrt{x}}$, $0\le x\le \frac{\pi^2}{4}$, око Ox осе.

1 пр.

$$V_x = \pi\int_a^b y^2\,dx, \quad y^2=\sqrt{x}\,\sin\sqrt{x}$$ 5

$$V_x = \pi\int_0^{\frac{\pi^2}{4}}\sqrt{x}\,\sin\sqrt{x}\,dx = \left[\begin{array}{l}\sqrt{x}=t \Rightarrow x=t^2\\ \qquad dx=2t\,dt\end{array}\right] = 2\pi\int_0^{\frac{\pi}{2}} t^2\sin t\,dt$$ 5

$$I=\int t^2\sin t\,dt = -\cos t\cdot t^2 + 2\int t\cos t\,dt = -t^2\cos t + 2t\sin t -$$

$$\left[\begin{array}{ll}u=t^2 & dv=\sin t\,dt\\ du=2t\,dt & v=-\cos t\end{array}\right]$$ 9 $$\left[\begin{array}{ll}u=t & dv=\cos t\,dt\\ du=dt & v=\sin t\end{array}\right]$$ 7

$$-2\int \sin t\,dt = -t^2\cos t + 2t\sin t + 2\cos t + C$$

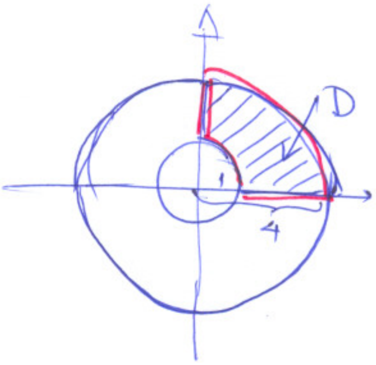
4 4

$$V_x = 2\pi\left(I\left(\frac{\pi}{2}\right)-I(0)\right) = 2\pi\left(0+\pi+0+0-0-2\right)$$

$$= 2\pi(\pi-2)$$ 1

$$\iint_D y e^{\sqrt{x^2+y^2}} \, dxdy =$$

$$D = \{(x,y) : 1 \leq x^2+y^2 \leq 16, \; x \geq 0, y \geq 0\}$$



$$= \iint_{D'} r\sin\varphi \cdot e^{\frac{r\cos\varphi}{r}} \cdot r\,dr\,d\varphi$$

$$= \iint_{D'} r^2 \sin\varphi \, e^{\cos\varphi} \, dr\,d\varphi \quad 4$$

$$= \int_1^4 dr \int_0^{\pi/2} r^2 \sin\varphi \cdot e^{\cos\varphi} \, d\varphi \quad 3$$

$$= \int_1^4 r^2 dr, \int_0^{\pi/2} \sin\varphi \, e^{\cos\varphi} \, d\varphi$$

$$= \int_1^4 r^2 dr \int_1^0 -e^t \, dt$$

$$= \int_1^4 r^2 dr \int_0^1 e^t \, dt$$

$$= \int_1^4 r^2 dr \left(e^t \Big|_0^1\right) \quad 2$$

$$= (e-1) \cdot \left(\frac{r^3}{3}\Big|_1^4\right) = (e-1) \cdot \frac{64-1}{3} = 21(e-1)$$

2+4 4

ПОЛАРНЕ КООРД.

$x = r\cos\varphi$

$y = r\sin\varphi$

$J = r$

D': $1 \leq r \leq 4$, $0 \leq \varphi \leq \pi/2$

6

СМЕНА:

$t = \cos\varphi$

$dt = -\sin\varphi \, d\varphi$

5