2. пр. $\int \frac{(9\cos x - 10 - 3\cos^2 x)\cdot \sin x}{(\cos x + 2)(\cos^2 x - 4\cos x + 8)}\,dx$

1.
$\begin{cases} \cos x = t \\ -\sin x\,dx = dt \end{cases}$ ⑤

$= \int \frac{(9t - 10 - 3t^2)\cdot(-1)\,dt}{(t+2)(t^2-4t+8)}$

$= \int \frac{(3t^2 - 9t + 10)\,dt}{(t+2)(t^2-4t+8)}$

$= \int \frac{2}{t+2}\,dt + \int \frac{t-3}{t^2-4t+8}\,dt$

$= \int \frac{2}{t+2}\,dt + \int \frac{t-2-1}{t^2-4t+8}\,dt = 2\ln|t+2| + \int \frac{t-2}{t^2-4t+8}\,dt - \int \frac{dt}{t^2-4t+8}$ ①

$= 2\ln|t+2| + \int \frac{\frac{1}{2}\,dy}{y}$ ③ $- \int \frac{dt}{(t-2)^2+2^2}$

$= 2\ln|t+2| + \frac{1}{2}\ln|y|$ ② $- \frac{1}{4}\int \frac{dt}{(\frac{t-2}{2})^2+1}$

$= 2\ln|t+2| + \frac{1}{2}\ln|y| - \frac{1}{4}\int \frac{2\,dz}{z^2+1}$

$= 2\ln|t+2| + \frac{1}{2}\ln(t^2-4t+8) - \frac{1}{2}\operatorname{arctg} z$ ③ $+C$

$= 2\ln|\cos x+2| + \ln\sqrt{\cos^2 x - 4\cos x + 8} - \frac{1}{2}\operatorname{arctg}\left(\frac{\cos x-2}{2}\right) + C$ ②

2.
$\begin{cases} t^2-4t+8 = y \\ dy = 2(t-2)\,dt \end{cases}$

3.
$\frac{t-2}{2} = z$

$\frac{dt}{2} = dz$

$dt = 2\,dz$

$\frac{3t^2-9t+10}{(t+2)(t^2-4t+8)} = \frac{A}{t+2} + \frac{Bt+C}{t^2-4t+8}$ ⑤

$= \frac{At^2-4At+8A+Bt^2+Ct+2Bt+2C}{(t+2)(t^2-4t+8)}$

$= \frac{(A+B)t^2+(-4A+2B+C)t+8A+2C}{(t+2)(t^2-4t+8)}$

$A+B=3$
$-4A+2B+C=-9$
$8A+2C=10 \;/\cdot(-\frac{1}{2})$ ↑+

$A+B=3$
$-8A+2B=-14$ ⟶

⑧ $A=2$, $B=1$, $C=-3$

•) Израчунати површину тела насталог ротирањем криве 2тр. $y = \ln x - \frac{x^2}{8}$, $\sqrt{e} \le x \le e$, око Ox осе.

$$P_x = 2\pi \int_a^b y\sqrt{1+y'^2}\,dx \quad (5)$$

$$y' = \frac{1}{x} - \frac{x}{4} \Rightarrow 1 + y'^2 = 1 + \frac{1}{x^2} + \frac{x^2}{16} - \frac{1}{2} = \frac{1}{x^2} + \frac{x^2}{16} + \frac{1}{2} = \left(\frac{1}{x} + \frac{x}{4}\right)^2 \quad (7)$$

$$P_x = 2\pi \int_{\sqrt{e}}^{e} \left(\ln x - \frac{x^2}{8}\right)\left(\frac{1}{x} + \frac{x}{4}\right) dx = 2\pi \int_{\sqrt{e}}^{e} \left(\frac{\ln x}{x} + \frac{1}{4}x\ln x - \frac{x}{8} - \frac{x^3}{32}\right) dx = 2\pi\left(-\frac{x^2}{16}\Big|_{\sqrt{e}}^{e} - \frac{x^4}{128}\Big|_{\sqrt{e}}^{e}\right) + \int_{\sqrt{e}}^{e} \ln x\, d(\ln x) + \frac{1}{4}\int_{\sqrt{e}}^{e} x\ln x\,dx \quad (4)\ (2)\ (2)\ (8)$$

$$= 2\pi\left(-\frac{e^2}{16} + \frac{e}{16} - \frac{e^4}{128} + \frac{e^2}{128} + \ln^2 x\Big|_{\sqrt{e}}^{e} + \right.$$

$$\left. + \frac{1}{4}\left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)\Big|_{\sqrt{e}}^{e}\right) = 2\pi\left(\frac{-8e^2 + 8e - e^4 + e^2}{128} + 1 - \frac{1}{4}\right.$$

$$\left. + \frac{1}{4}\left(\frac{e^2}{2} - \frac{e^2}{4}\right) - \frac{1}{4}\left(\frac{e}{4} - \frac{e}{4}\right)\right) = \frac{\pi}{64}\left(e^2 + 8e - e^4 + 96\right)$$

$\frac{e^2}{16}$ (4) (4) $= 0$ (1)

$\left[u = \ln x \quad dv = x\,dx \quad du = \frac{dx}{x} \quad v = \frac{x^2}{2}\right]$

2. TP.

$$\iint_D (3x+y)\cos\left(\pi(x-3y)\right)dxdy$$

$$= \iint_{D'} u\cdot\cos(\pi v)\cdot\left|-\frac{1}{10}\right| du\,dv \quad (2)$$

$$= \int_1^3 du \int_0^{1/4} u\cos(\pi v)\cdot\frac{1}{10}\,dv \quad (3)$$

$$= \frac{1}{10}\int_1^3 u\,du \int_0^{1/4}\cos(\pi v)\,dv$$

$$= \frac{1}{10}\cdot\left(\frac{u^2}{2}\Big|_1^3\right)\cdot\left(\frac{1}{\pi}\cdot\sin(\pi v)\Big|_0^{1/4}\right)$$

(3) (2)

$$= \frac{1}{10\pi}\cdot\frac{9-1}{2}\cdot\left(\sin\pi/4 - \sin 0\right)$$

(4) (60)

$$= \frac{8}{20\pi}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{5\pi} \quad (1)$$

D: $y = -3x+3$

$y = -3x+1$

$y = x/3$

$y = x/3 - \frac{1}{12}$

$y+3x = u$

$x-3y = v$ (5)

$$x = \frac{3u+v}{10}$$

$$y = \frac{u-3v}{10}$$

$$J = \begin{vmatrix} \frac{3}{10} & \frac{1}{10} \\ \frac{1}{10} & -\frac{3}{10} \end{vmatrix}$$

$$= -\frac{9}{100} - \frac{1}{100}$$

$$= -\frac{1}{10} \quad (2)$$

D': $1 \le u \le 3$

$0 \le v \le \frac{1}{4}$