3.
Пр.

$$\int \frac{(3\sin^2 x + 2\sin x + 11)\cdot\cos x}{(\sin x - 1)(\sin^2 x + 2\sin x + 5)}\,dx$$

1.
$$\left\ulcorner \begin{aligned} \sin x &= t \\ dt &= \cos x\,dx \end{aligned} \right\lrcorner$$
5

$$= \int \frac{3t^2 + 2t + 11}{(t-1)(t^2+2t+5)}\,dt$$

$$= \int \frac{2}{t-1}\,dt + \int \frac{t-1}{t^2+2t+5}\,dt$$

2.
$$\left\ulcorner \begin{aligned} t^2+2t+5 &= y \\ (2t+2)\,dt &= dy \end{aligned} \right\lrcorner$$

$$= 2\ln|t-1| + \int \frac{t+1-2}{t^2+2t+5}\,dt$$
3

$$= 2\ln|t-1| + \int \frac{t+1}{t^2+2t+5}\,dt - 2\int \frac{dt}{(t+1)^2+2^2}$$
1

3.
$$\left\ulcorner \begin{aligned} \frac{t+1}{2} &= z \\ dt &= 2\,dz \end{aligned} \right\lrcorner$$

$$= 2\ln|t-1| + \int \frac{\frac{1}{2}\,dy}{y} - 2\cdot\frac{1}{4}\cdot\int \frac{dt}{\left(\frac{t+1}{2}\right)^2+1}$$
3

$$= 2\ln|t-1| + \frac{1}{2}\ln|y| - \frac{1}{2}\int \frac{2\,dz}{z^2+1}$$
2

$$= 2\ln|\sin x - 1| + \ln\sqrt{t^2+2t+5} - \operatorname{arctg} z + C$$
3

$$= 2\ln|\sin x - 1| + \ln\sqrt{\sin^2 x + 2\sin x + 5} - \operatorname{arctg}\left(\frac{\sin x + 1}{2}\right) + C$$
3 2

$$\frac{3t^2+2t+11}{(t-1)(t^2+2t+5)} = \frac{A}{t-1} + \frac{Bt+C}{t^2+2t+5}$$
5

$$= \frac{At^2 + 2At + 5A + Bt^2 + Ct - Bt - C}{(t-1)(t^2+2t+5)}$$

$$= \frac{(A+B)t^2 + (2A-B+C)t + 5A - C}{(t-1)(t^2+2t+5)}$$

$$\begin{aligned} A+B &= 3 \\ 2A-B+C &= 2 \\ 5A-C &= 11 \end{aligned} \Big\} +$$

$$\left.\begin{aligned} A+B &= 3 \\ 7A-B &= 13 \end{aligned}\right\} \longrightarrow \begin{aligned} A &= 2 \\ B &= 1 \\ C &= -1 \end{aligned}$$
8

Израчунати запремину тела насталог ротирањем криве $y = \sqrt{\sqrt{x}\, e^{\sqrt{x}}}$, $0 \le x \le 1$, око Ox осе.

3тр.

$$V_x = \pi \int_a^b y^2 dx, \quad y^2 = \sqrt{x}\, e^{\sqrt{x}}$$ 5

$$V_x = \pi \int_0^1 \sqrt{x}\, e^{\sqrt{x}} dx = \left[\begin{array}{l} \sqrt{x} = t \Rightarrow x = t^2 \\ dx = 2t\,dt \end{array} \right] = 2\pi \int_0^1 t^2 e^t dt$$ 5

$$I = \int t^2 e^t dt = t^2 e^t - 2\int t e^t dt = t^2 e^t - 2te^t + 2\int e^t dt$$

$$\left[\begin{array}{ll} u = t^2 & dv = e^t dt \\ du = 2t\,dt & v = e^t \end{array} \right] \quad \left[\begin{array}{ll} u = t & dv = e^t dt \\ du = dt & v = e^t \end{array} \right]$$ 9 7

$$= t^2 e^t - 2te^t + 2e^t + C$$

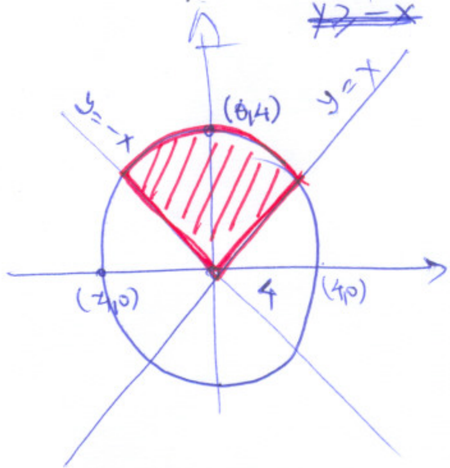
$$V_x = 2\pi (I(1) - I(0)) = 2\pi (e - \cancel{2e} + \cancel{2e} - 0 + 0 - 2)$$ 4 4

$$= 2\pi (e - 2)$$ 1

смена 5

3. пр.

$$\iint_D \arcsin \frac{y}{\sqrt{x^2+y^2}}\,dx\,dy \qquad D = \{(x,y):\ x^2+y^2 \le 16,\ \cancel{-x \le y \le x},\ -y \le x \le y,\ \cancel{y \ge -x}\}$$



ПОЛАРНЕ КООРД.

$$\left.\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \\ J &= r \end{aligned}\right\}\ 5$$

$$D': \left.\begin{aligned} 0 \le\ & r \le 4 \\ \pi/4 \le\ & \varphi \le 3\pi/4 \end{aligned}\right\}\ 3$$

$$= \iint_{D'} \arcsin\left(\frac{\cancel{r}\sin\varphi}{\cancel{r}}\right) r\,dr\,d\varphi \quad 5$$

$$= \iint_{D'} \varphi \cdot r\,dr\,d\varphi \quad 4$$

$$= \int_0^4 r\,dr \int_{\pi/4}^{3\pi/4} \varphi\,d\varphi$$

$$= \left(\frac{r^2}{2}\Big|_0^4\right) 3 \cdot \left(\frac{\varphi^2}{2}\Big|_{\pi/4}^{3\pi/4}\right) 3$$

$$= \left(\frac{16-0}{2}\right) 4 \cdot \left(\frac{\frac{9}{16}\pi^2 - \frac{1}{16}\pi^2}{2}\right) 4$$

$$= 4\cdot\pi^2\left(\frac{9-1}{16}\right)$$

$$= 2\pi^2 \quad 1$$