пр. $\displaystyle\int \frac{3e^{3x} - e^{2x} + 4e^{x}}{(e^{x}+1)(e^{2x}-2e^{x}+5)}\,dx$

1. СМЕНА

$\ulcorner\ t = e^{x}$

$dt = e^{x}dx\ \lrcorner$ 5

$$= \int \frac{(3e^{2x} - e^{x} + 4)\cdot e^{x}}{(e^{x}+1)(e^{2x}-2e^{x}+5)}\,dx$$

$$= \int \frac{3t^2 - t + 4}{(t+1)(t^2-2t+5)}\,dt$$

2. СМЕНА

$\ulcorner\ t^2-2t+5 = y$

$(2t-2)\,dt = dy\ \lrcorner$

$$= \int \frac{1}{t+1}\,dt + \int \frac{2t-1}{t^2-2t+5}\,dt$$

$$= \int \frac{1}{t-1}\,dt + \int \frac{2t-2+1}{t^2-2t+5}\,dt$$

$$= \int \frac{1}{t-1}\,dt + \int \frac{2t-2}{t^2-2t+5}\,dt \ \text{1} + \int \frac{1}{(t-1)^2+2^2}\,dt$$

3. СМЕНА

$\ulcorner\ \frac{t-1}{2} = z$

$dt = 2dz\ \lrcorner$

$$= \ln|t-1| \ \text{3} + \int \frac{dy}{y} \ \text{3} + \int \frac{dt}{4\left(\left(\frac{t-1}{2}\right)^2+1\right)}$$

$$= \ln|e^{x}-1| + \ln|y| \ \text{2} + \frac{1}{4}\int \frac{2dz}{z^2+1}$$

$$= \ln|e^{x}-1| + \ln(t^2-2t+5) + \frac{1}{2}\operatorname{arctg} z \ \text{3} + C$$

$$= \ln|e^{x}-1| + \ln(e^{2x}-2e^{x}+5) + \frac{1}{2}\operatorname{arctg}\left(\frac{e^{x}-1}{2}\right) \ \text{3} + C \ \text{2}$$

$$\frac{3t^2-t+4}{(t+1)(t^2-2t+5)} = \frac{A}{t+1} + \frac{Bt+C}{t^2-2t+5} \quad \text{5}$$

$$= \frac{At^2 - 2At + 5A + Bt^2 + Ct + Bt + C}{(t+1)(t^2-2t+5)}$$

$$= \frac{(A+B)t^2 + (-2A+B+C)t + 5A + C}{(t+1)(t^2-2t+5)}$$

$$\begin{aligned} A+B &= 3 \\ -2A+B+C &= -1 \\ 5A+C &= 4 \end{aligned} \longrightarrow \begin{aligned} A+B &= 3 \\ 7A-B &= 5 \end{aligned} \longrightarrow \begin{aligned} A &= 1 \\ B &= 2 \\ C &= -1 \end{aligned} \quad \text{8}$$

4 гр.) Израчунати површину тела насталог ротирањем криве $y = x^2 - \frac{\ln x}{8}$, $\sqrt{e} \le x \le e$, око Ox осе.

$$P_x = 2\pi \int_a^b y\sqrt{1+y'^2}\,dx \quad 5$$

$$y' = 2x - \frac{1}{8x} \Rightarrow 1 + y'^2 = 1 + 4x^2 + \frac{1}{64x^2} - \frac{1}{2} = \left(2x + \frac{1}{8x}\right)^2 \quad 7$$

$$P_x = 2\pi \int_{\sqrt{e}}^{e} \left(x^2 - \frac{\ln x}{8}\right)\left(2x + \frac{1}{8x}\right) dx = 2\pi \int_{\sqrt{e}}^{e} \Big(2x^3 + \frac{x}{8} - \frac{1}{4}x\ln x$$

$$- \frac{1}{64}\frac{\ln x}{x}\Big) dx = 2\pi\left(\frac{x^4}{2}\Big|_{\sqrt{e}}^{e} + \frac{x^2}{16}\Big|_{\sqrt{e}}^{e} - \frac{1}{4}\int_{\sqrt{e}}^{e} x\ln x\,dx - \frac{1}{64}\int_{\sqrt{e}}^{e} \ln x\,d(\ln x)\right)$$

2 2 2 8 4

$$= 2\pi\left(\frac{e^4}{2} - \frac{e^2}{2} + \frac{e^2}{16} - \frac{e}{16} - \frac{1}{4}\left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)\Big|_{\sqrt{e}}^{e}\right.$$

$$\left[\begin{array}{ll} u = \ln x & dv = x\,dx \\ du = \frac{dx}{x} & v = \frac{x^2}{2} \end{array}\right]$$

$$\left. - \frac{1}{64}\ln^2 x\Big|_{\sqrt{e}}^{e}\right) =$$

$$= 2\pi\left(\frac{8e^4 - 8e^2 + e^2 - e}{16} - \underbrace{\frac{1}{4}\left(\frac{e^2}{2} - \frac{e^2}{4}\right)}_{-e^2/16} + \underbrace{\frac{1}{4}\left(\frac{e}{4} - \frac{e}{4}\right)}_{=0} - \frac{1}{64} + \frac{1}{256}\right)$$

4 4

$$= \frac{\pi}{128}\left(128e^4 - 128e^2 - 16e - 3\right)$$

1

4\.

пр. $\iint_D (x-2y)\sin(\pi(2x+y))\,dx\,dy$

$= \iint_{D'} v\cdot \sin\pi u \cdot \left|-\frac{1}{5}\right|\cdot du\,dv$ 3

$= \frac{1}{5}\int_{-6}^{-4} dv \int_0^{1/2} v\cdot\sin(\pi u)\,du$ 3

$= \frac{1}{5}\int_{-6}^{-4} v\,dv \int_0^{1/2} \sin(\pi u)\,du$

$= \frac{1}{5}\cdot\left(\frac{v^2}{2}\Big|_{-6}^{-4}\right)\cdot\left(\frac{1}{\pi}\cdot(-\cos(\pi u))\Big|_0^{1/2}\right)$ 2 3

$= \frac{1}{5}\cdot\frac{16-36}{2}\cdot\frac{1}{\pi}\cdot\left(-\underbrace{\cancel{\cos\pi/2}}_{0}+\underbrace{\cos 0}_{1}\right)$ 4 6

$= \frac{-20}{10}\cdot\frac{1}{\pi} = -\frac{2}{\pi}$ 1

$D: \left.\begin{aligned} y &= \frac{x}{2}+2 \\ y &= \frac{x}{2}+3 \end{aligned}\right\} \rightarrow \begin{aligned} y-\frac{x}{2} &= 2 \\ y-\frac{x}{2} &= 3 \end{aligned}$

$\left.\begin{aligned} y &= -2x \\ y &= -2x+\frac{1}{2} \end{aligned}\right\} \begin{aligned} y+2x &= 0 \\ y+2x &= \frac{1}{2} \end{aligned}$

СМЕНЕ:

$\boxed{\begin{aligned} u &= 2x+y \\ v &= x-2y \end{aligned}}$ 5

$2u+v = 4x+x = 5x$

$\boxed{\begin{aligned} x &= \frac{2u+v}{5} \\ y &= \frac{u-2v}{5} \end{aligned}}$

$J = \begin{vmatrix} \frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{vmatrix} = -\frac{4}{25}-\frac{1}{25} = -\frac{1}{5}$ 2

$D': \quad 0\le u\le \frac{1}{2}$

$-6\le v\le -4$